Exercise 5C

1 a $m = 2, (x_1, y_1) = (2, 5)$

$y - y_1 = m(x - x_1)$
$y - 5 = 2(x - 2)$
$y - 5 = 2x - 4$
$y = 2x + 1$

b $m = 3, (x_1, y_1) = (-2, 1)$

$y - y_1 = m(x - x_1)$
$y - 1 = 3(x - (-2))$
$y - 1 = 3(x + 2)$
$y - 1 = 3x + 6$
$y = 3x + 7$

c $m = -1, (x_1, y_1) = (3, -6)$

$y - y_1 = m(x - x_1)$
$y - (-6) = -1(x - 3)$
$y + 6 = -x + 3$
$y = -x - 3$

d $m = -4, (x_1, y_1) = (-2, -3)$

$y - y_1 = m(x - x_1)$
$y - (-3) = -4(x - (-2))$
$y + 3 = -4(x + 2)$
$y + 3 = -4x - 8$
$y = -4x - 11$

e $m = \frac{1}{2}, (x_1, y_1) = (-4, 10)$

$y - y_1 = m(x - x_1)$
$y - 10 = \frac{1}{2}(x - (-4))$
$y - 10 = \frac{1}{2}(x + 4)$
$y - 10 = \frac{1}{2}x + 2$
$y = \frac{1}{2}x + 12$

f $m = -\frac{2}{3}, (x_1, y_1) = (-6, -1)$

$y - y_1 = m(x - x_1)$
$y - (-1) = -\frac{2}{3}(x - (-6))$
$y + 1 = -\frac{2}{3}(x + 6)$
$y + 1 = -\frac{2}{3}x - 4$
$y = -\frac{2}{3}x - 5$

g $m = 2, (x_1, y_1) = (a, 2a)$

$y - y_1 = m(x - x_1)$
$y - 2a = 2(x - a)$
$y - 2a = 2x - 2a$
$y = 2x$

h $m = -\frac{1}{2}, (x_1, y_1) = (-2b, 3b)$

$y - y_1 = m(x - x_1)$
$y - 3b = -\frac{1}{2}(x - (-2b))$
$y - 3b = -\frac{1}{2}(x + 2b)$
$y - 3b = -\frac{1}{2}x - b$
$y = -\frac{1}{2}x - b + 3b$
$y = -\frac{1}{2}x + 2b$

2 a $(x_1, y_1) = (2, 4), (x_2, y_2) = (3, 8)$

The gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{8 - 4}{3 - 2}$
$= 4$

The equation is $y - y_1 = m(x - x_1)$
$y - 4 = 4(x - 2)$
$y - 4 = 4x - 8$
$y = 4x - 4$

b $(x_1, y_1) = (0, 2), (x_2, y_2) = (3, 5)$

The gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5 - 2}{3 - 0}$
$= \frac{3}{3}$
$= 1$

The equation is $y - y_1 = m(x - x_1)$
$y - 2 = 1(x - 0)$
$y - 2 = x$
$y = x + 2$

2 c $(x_1, y_1) = (-2, 0), (x_2, y_2) = (2, 8)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{8-0}{2-(-2)}$$
$$= \frac{8}{4}$$
$$= 2$$

The equation is $y - y_1 = m(x - x_1)$

$y - 0 = 2(x - (-2))$

$y = 2(x + 2)$

$y = 2x + 4$

d $(x_1, y_1) = (5, -3), (x_2, y_2) = (7, 5)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{5-(-3)}{7-5}$$
$$= \frac{8}{2}$$
$$= 4$$

The equation is $y - y_1 = m(x - x_1)$

$y - (-3) = 4(x - 5)$

$y + 3 = 4x - 20$

$y = 4x - 23$

e $(x_1, y_1) = (3, -1), (x_2, y_2) = (7, 3)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{3-(-1)}{7-3}$$
$$= \frac{4}{4}$$
$$= 1$$

The equation is $y - y_1 = m(x - x_1)$

$y - (-1) = 1(x - 3)$

$y + 1 = x - 3$

$y = x - 4$

f $(x_1, y_1) = (-4, -1), (x_2, y_2) = (6, 4)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{4-(-1)}{6-(-4)}$$
$$= \frac{5}{10}$$
$$= \frac{1}{2}$$

The equation is $y - y_1 = m(x - x_1)$

$y - (-1) = \frac{1}{2}(x - (-4))$

$y + 1 = \frac{1}{2}(x + 4)$

$y + 1 = \frac{1}{2}x + 2$

$y = \frac{1}{2}x + 1$

g $(x_1, y_1) = (-1, -5), (x_2, y_2) = (-3, 3)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{3-(-5)}{-3-(-1)}$$
$$= \frac{8}{-2}$$
$$= -4$$

The equation is $y - y_1 = m(x - x_1)$

$y - (-5) = -4(x - (-1))$

$y + 5 = -4(x + 1)$

$y + 5 = -4x - 4$

$y = -4x - 9$

h $(x_1, y_1) = (-4, -1), (x_2, y_2) = (-3, -9)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-9-(-1)}{-3-(-4)}$$
$$= \frac{-8}{1}$$
$$= -8$$

2 h The equation is $y - y_1 = m(x - x_1)$

$y - (-1) = -8(x - (-4))$

$y + 1 = -8(x + 4)$

$y + 1 = -8x - 32$

$y = -8x - 33$

i $(x_1, y_1) = (\frac{1}{3}, \frac{2}{5}), (x_2, y_2) = (\frac{2}{3}, \frac{4}{5})$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{\frac{4}{5} - \frac{2}{5}}{\frac{2}{3} - \frac{1}{3}}$$

$$= \frac{\frac{2}{5}}{\frac{1}{3}}$$

$$= \frac{2}{5} \times 3$$

$$= \frac{6}{5}$$

The equation is $y - y_1 = m(x - x_1)$

$y - \frac{2}{5} = \frac{6}{5}(x - \frac{1}{3})$

$y - \frac{2}{5} = \frac{6}{5}x - \frac{2}{5}$

$y = \frac{6}{5}x$

j $(x_1, y_1) = (-\frac{3}{4}, \frac{1}{7}), (x_2, y_2) = (\frac{1}{4}, \frac{3}{7})$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{\frac{3}{7} - \frac{1}{7}}{\frac{1}{4} - (-\frac{3}{4})}$$

$$= \frac{\frac{2}{7}}{1}$$

$$= \frac{2}{7}$$

The equation is $y - y_1 = m(x - x_1)$

$y - \frac{1}{7} = \frac{2}{7}(x - (-\frac{3}{4}))$

$y - \frac{1}{7} = \frac{2}{7}x + \frac{3}{4}$

$y = \frac{2}{7}x + \frac{3}{4} + \frac{1}{7}$

$y = \frac{2}{7}x + \frac{5}{14}$

3 Line passes through (7, 2) and (9, −8).

$(x_1, y_1) = (7, 2), (x_2, y_2) = (9, -8)$

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-8 - 2}{9 - 7}$$

$$= \frac{-10}{2}$$

$$= -5$$

The equation is $y - y_1 = m(x - x_1)$

$y - 2 = -5(x - 7)$

$y - 2 = -5x + 35$

$y + 5x - 37 = 0$

4 For the equation of AB:

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{0 - 5}{-2 - 3}$$

$$= \frac{-5}{-5}$$

$$= 1$$

The equation is $y - y_1 = m(x - x_1)$

$y - 5 = 1(x - 3)$

$y - 5 = x - 3$

$y = x + 2$ or $y - x - 2 = 0$

For the equation of AC:

The gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1 - 5}{4 - 3}$$

$$= \frac{-6}{1}$$

$$= -6$$

The equation is $y - y_1 = m(x - x_1)$

$y - 5 = -6(x - 3)$

$y - 5 = -6x + 18$

$y = -6x + 23$ or $6x + y - 23 = 0$

For the equation of BC:

$(x_1, y_1) = (-2, 0), (x_2, y_2) = (4, -1)$

4 The gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$m = \frac{-1-0}{4-(-2)}$$
$$= -\frac{1}{6}$$

The equation is $y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{6}(x - (-2))$

$y = -\frac{1}{6}(x + 2)$

$y = -\frac{1}{6}x - \frac{1}{3} \Rightarrow 6y = -x - 2$

or $\frac{1}{6}x + y + \frac{1}{3} = 0 \Rightarrow x + 6y + 2 = 0$

5 Line through $(a, 4)$ and $(3a, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-4}{3a-a}$$
$$= -\frac{1}{2a}$$

$x + 6y + c = 0$

$6y = -x - c$

$y = -\frac{1}{6}x - \frac{1}{6}c$

$m = -\frac{1}{6} = -\frac{1}{2a}$, so $a = 3$.

As $a = 3$, $(a, 4)$ is the point $(3, 4)$.

Substituting $x = 3$ and $y = 4$ into $y = -\frac{1}{6}x - \frac{1}{6}c$ to find c:

$4 = -\frac{1}{6}(3) - \frac{1}{6}c$

$24 = -3 - c$

$c = -27$

$a = 3, c = -27$

6 Line through $(7a, 5)$ and $(3a, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{3-5}{3a-7a}$$
$$= \frac{-2}{-4a}$$
$$= \frac{1}{2a}$$

6 So $y - y_1 = m(x - x_1)$

$$y - 5 = \frac{1}{2a}(x - 7a)$$
$$2ay - 10a = x - 7a$$
$$x - 2ay + 3a = 0$$

But the equation of line l is:

$x + by - 12 = 0$

Therefore, $3a = -12$, $a = -4$.

Using the coefficients of x:

$-2a = b$

$a = -4, b = 8$

Challenge

a $m = \frac{y_2 - y_1}{x_2 - x_1}$

b Using $y - y_1 = m(x - x_1)$:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$
$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

c Passes through $(-8, 4)$ and $(-1, 7)$.

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$
$$\frac{y-4}{7-4} = \frac{x-(-8)}{(-1)-(-8)}$$
$$\frac{y-4}{3} = \frac{x+8}{7}$$
$$y - 4 = \frac{3(x+8)}{7}$$
$$y = \frac{3}{7}x + \frac{24}{7} + 4$$
$$y = \frac{3}{7}x + \frac{52}{7}$$